

Modeling Spatially Variant Processes in Serpentine Activated Sludge Reactors

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*Environmental
Engineering Process Analysis*

Perspective

Aeration continues to be a costly operational aspect of activated sludge treatment.

From both cost and sustainability considerations- increased efficiency of aeration systems is highly desired.

Operations staff desire efficient, robust systems that can be “optimized”.

Currently no published models are available specifically addressing the spatially variant processes within activated sludge reactors.

The microchip, PC and convenient software permit modeling of many systems we've thought too onerous to attempt.

To illustrate this modeling approach:

We investigate a fine-bubble aeration system for a high-rate AS reactor performing BOD reduction and nitrification, arranged to resemble a plug-flow (a.k.a. – a serpentine) reactor.

We'll use modeling results to:

- Lay out the system of fine-bubble diffusers
- Determine the “turn-down” capacity of the system

Available reactor models

Ideal:

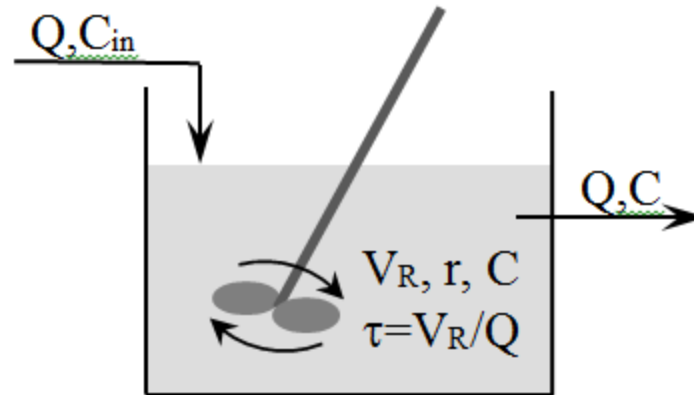
- Completely-Mixed Flow Reactor (CMFR)
- Plug-Flow Reactor (PFR)

Non-ideal (real):

- CMFRs (Tanks) in Series (TiS)
- Segregated Flow (SF)

Completely-Mixed Flow Reactor (CMFR)

(Continuously-Stirred Tank Reactor (CSTR))



All parameters have identical abundances in
the effluent and reactor itself

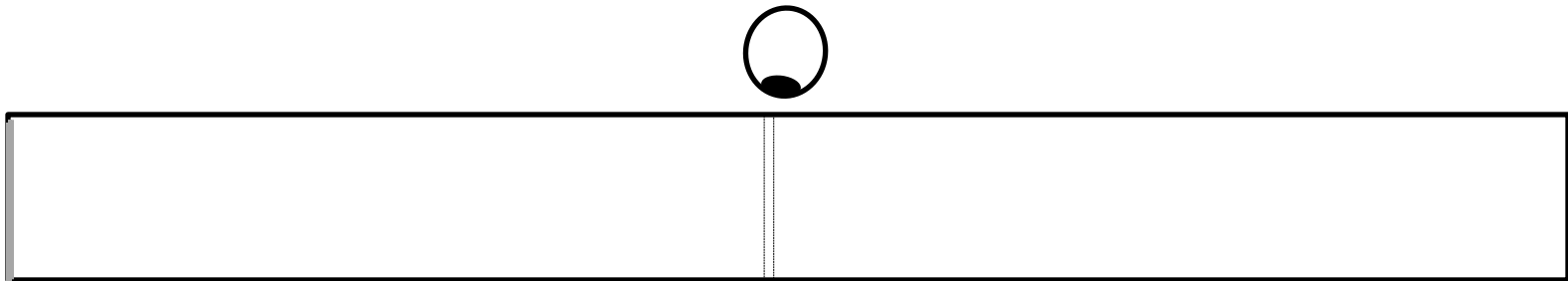
Steady-state mass balances yield algebraic equations

Transient mass balances yield ordinary differential equations

No spatial variation in any parameters

The Plug Flow Reactor (PFR)

Elements of fluid enter, are instantaneously distributed across the entire cross-section of the reactor and pass through the reactor in order, never mixing with elements ahead or behind.

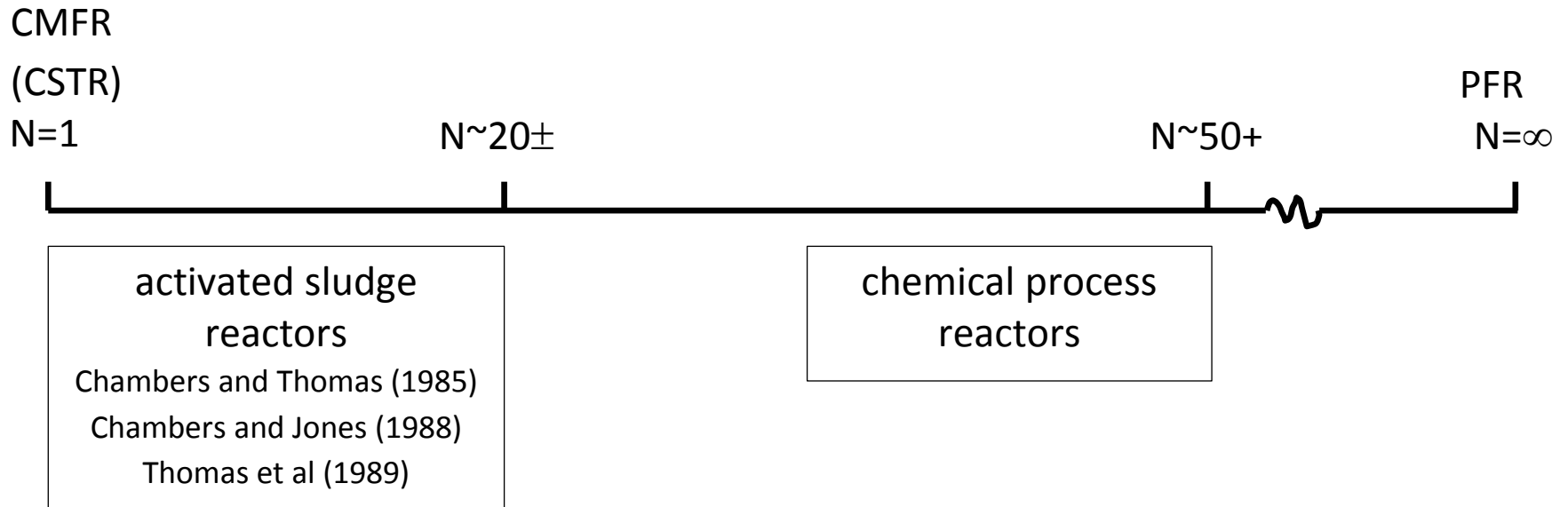


We choose an arbitrary position along the reactor and observe the progress of our process between the inlet and our observation point. (Eulerian view, PFR)

Obstacle for employment – steady state mass balance on a PFR leads to an ordinary differential equation, transient case leads to a partial differential equation

Allows for modeling of spatial variations

Real Reactor Tanks in Series number line



for $N > 20$, errors with use of the PFR model diminish greatly.

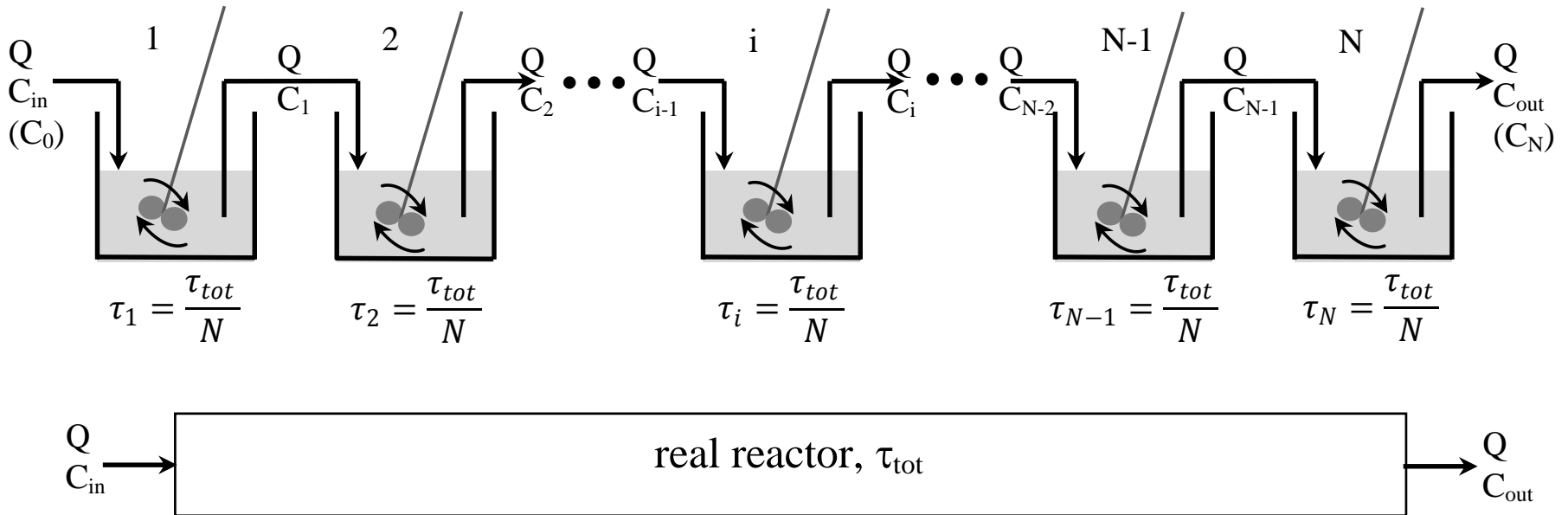
Which is probably why the ChEs, who developed non-ideal reactor theory have not had much interest in expanding/employing the theory over the past six decades.

Chambers, B and VK Thomas, Energy Saving by Optimisation of Activated Sludge Aeration, proceedings of 8th Symposium on Wastewater Treatment, Montreal Quebec, 1985.

Chambers, B and GL Jones, Optimisation and Uprating of Activated Sludge Plants by Efficient Process Design, Water Sci. Tech., 20, 1988.

Thomas, VK, B Chambers and W Dunn, Optimisation of Aeration Efficiency: A Design Procedure for Secondary Treatment using a Hybrid Aeration System, Water Sci. Tech., 21, 1989.

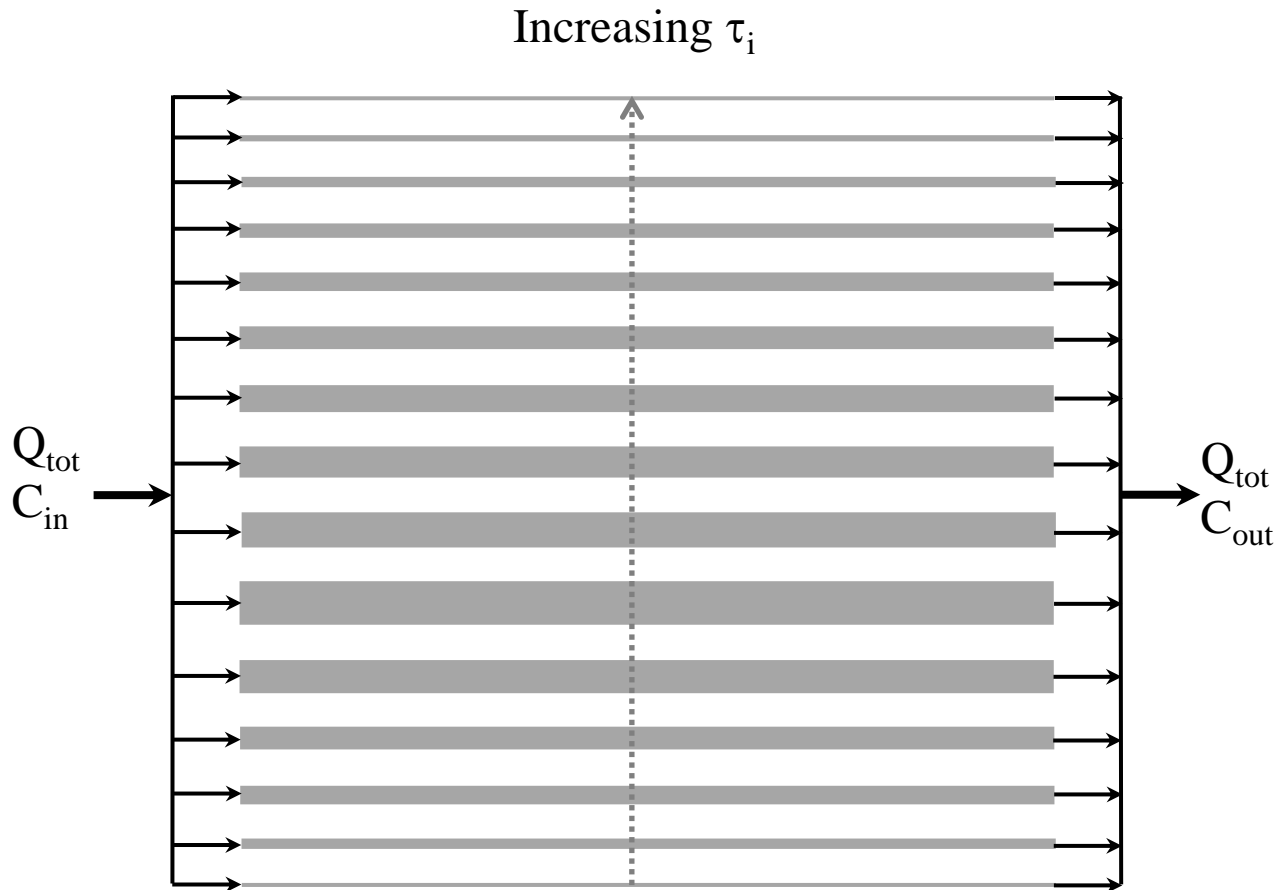
The CMFRs in Series Model



We simply solve for each reactor effluent in turn.

The Segregated Flow Model:

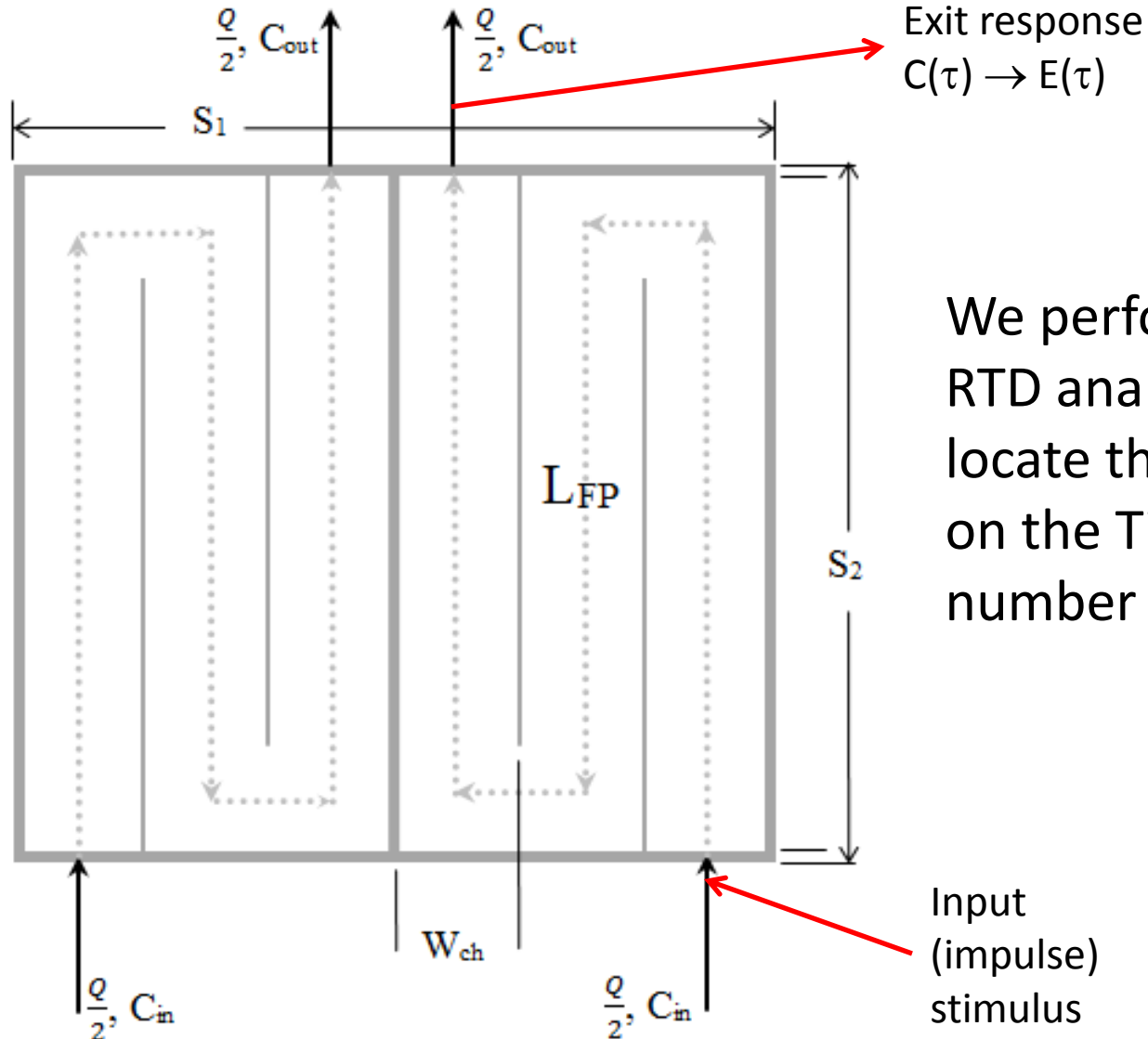
A set of parallel PFRs comprises the reactor



$$C_{out} = \int_{\tau_{min}}^{\tau_{max}} E(\tau) C(\tau) d\tau$$

Simply a flow-weighted value

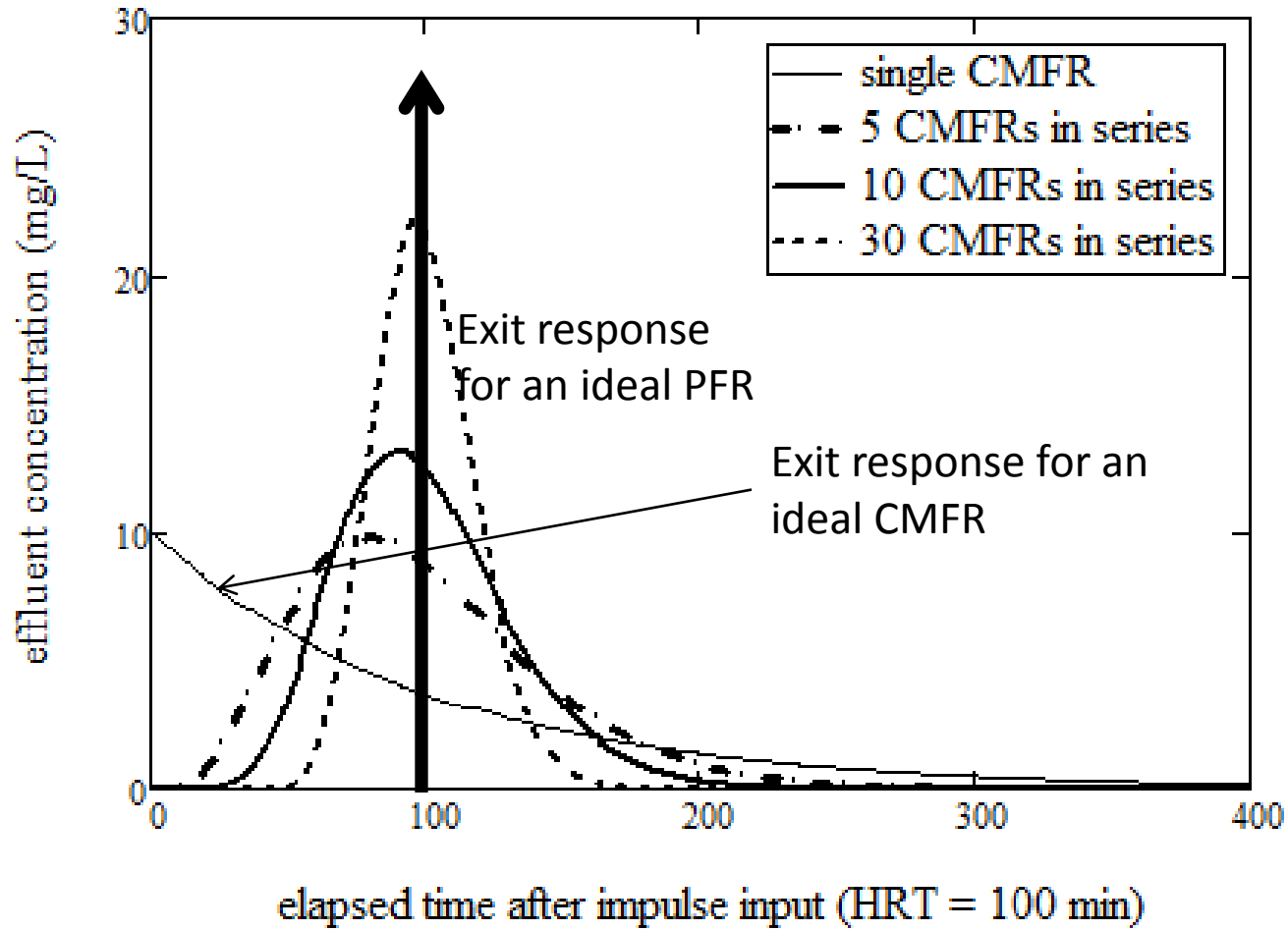
Our “serpentine” (plug-flow-like) reactor



We perform a RTD analysis to locate the reactor on the TiS number line.

Typical Exit Response Curves

discrete $C(\tau)$ vs τ data values

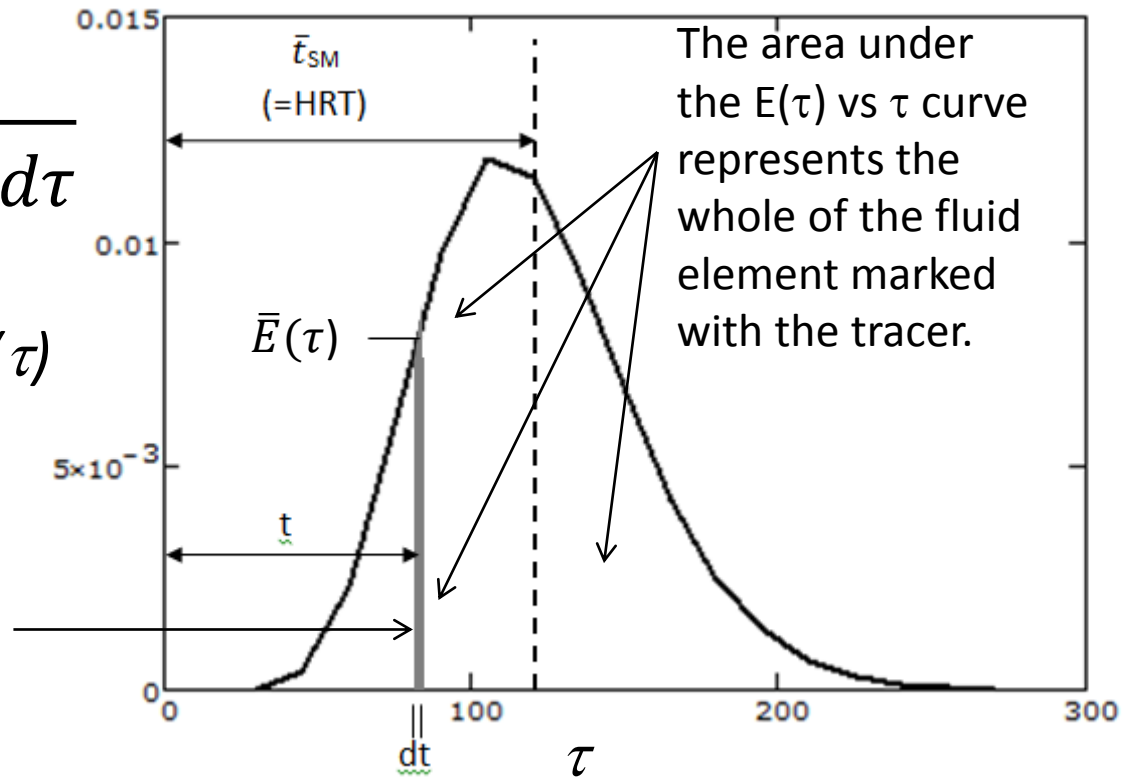


The RTD density distribution – $E(\tau)$ vs τ

$$E(\tau) = \frac{C(\tau)}{\int_{\tau_{min}}^{\tau_{max}} C(\tau) d\tau}$$

$E(\tau)$

The product $E(\tau) \cdot d\tau$ is the fraction of the marked fluid element exiting with residence time τ .



All subsequent fluid elements behave exactly as the marked one.

The remaining characterization can now be completed.

Parameters computed from RTD density

$$\bar{\tau}_{SM} = \int_{\tau_{min}}^{\tau_{max}} \tau \cdot E(\tau) d\tau \quad \sigma_t^2 = \int_{\tau_{min}}^{\tau_{max}} (\tau - \bar{\tau}_{SM})^2 \cdot E(\tau) d\tau$$

$$N = \frac{\bar{\tau}_{SM}^2}{\sigma_t^2} \quad \frac{1}{N} = 2 \frac{D_d}{v_S L_{FP}} - 2 \left(\frac{D_d}{v_S L_{FP}} \right)^2 \cdot \left(1 - e^{-\frac{v_S L_{FP}}{D_d}} \right)$$

Danckwerts (1953); Levenspiel (1972, 1999)

We must implicitly solve for D_d .

D_d is a constant for a specific aeration system, reactor configuration and air delivery rate

Danckwerts, PV, Continuous Flow Systems: distribution of Residence Times, *Chemical Engineering Science* 2, 1953.

Levenspiel, O, *Chemical Reaction Engineering*, 2nd ed., Wiley, NY, 1972.

Levenspiel, O, *Chemical Reaction Engineering*, 3rd ed., Wiley, NY, 1999.

We generalize the reactor characterization

L_{FP}, A_X
from reactor geometry

$$v_s = \frac{Q(1+R)}{A_X}$$

The recycle ratio, R ,
is a process control
parameter

$$N = \frac{1}{2 \frac{D_d}{v_s L_{FP}} - 2 \left(\frac{D_d}{v_s L_{FP}} \right)^2 \cdot \left(1 - e^{-\frac{v_s L_{FP}}{D_d}} \right)}$$

We can explicitly compute a new N corresponding with R

$$E(\theta) = \frac{N \cdot (N \cdot \theta)^{N-1}}{(N-1)!} e^{-N \cdot \theta}$$

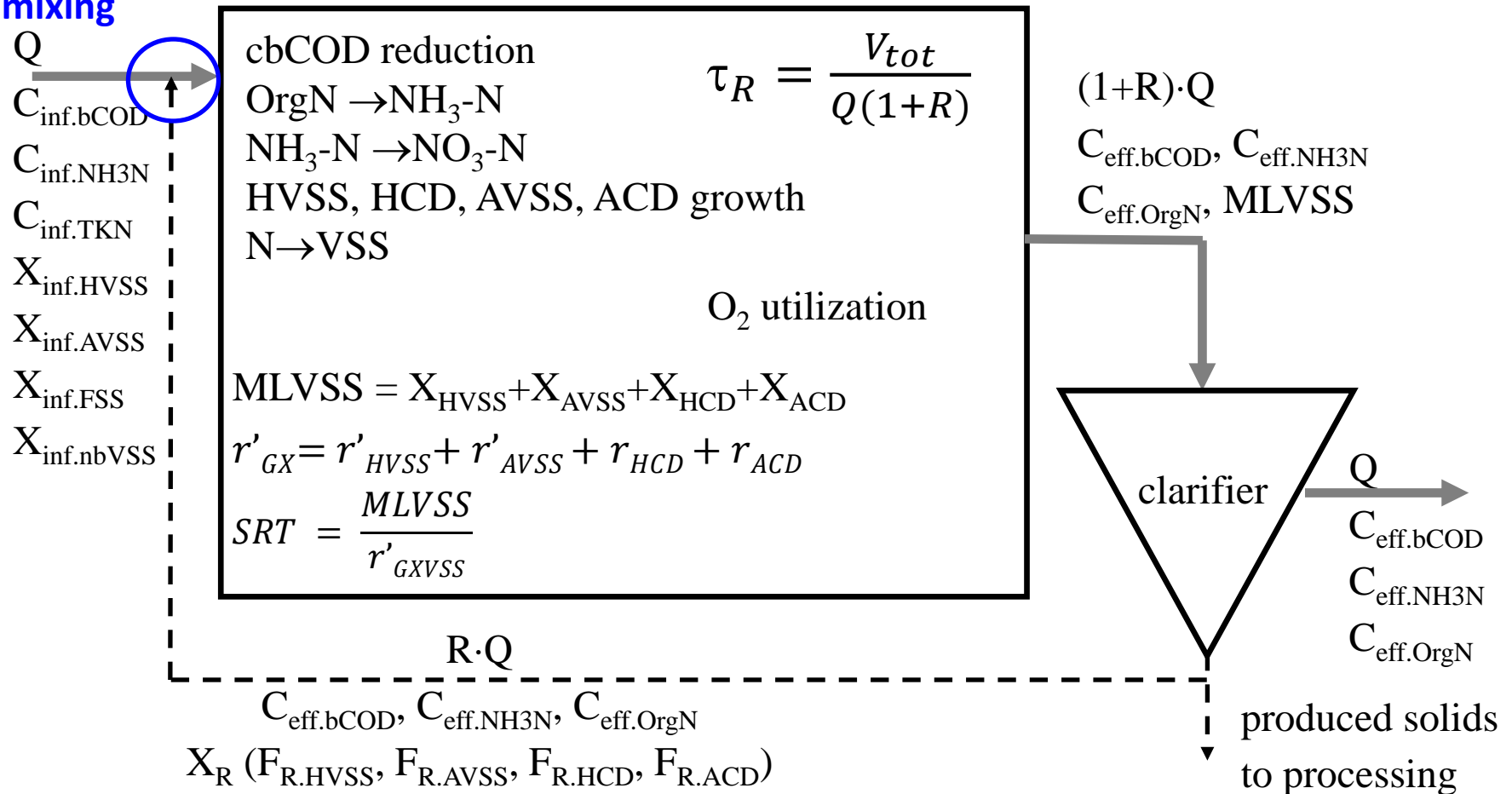
$$\theta = \frac{\tau}{\bar{\tau}_{SM}} \quad E(\tau) = \frac{E(\theta)}{\bar{\tau}_{SM}}$$

$E(\theta)$ and, hence, $E(\tau)$ are computed directly from N Levenspiel (1972, 1999)

The recycle reactor schematic and processes

Zero-
volume
mixing

Reactor: CMFR, PFR, TiS or SF



Typical kinetic and stoichiometric parameters

Tchobanoglous et al (2003)

$\mu_{\max.H}$	3.0	$\frac{\epsilon_{HVSS}}{\epsilon_{HVSS} \cdot d}$	$\mu_{\max.A}$	0.75	$\frac{\epsilon_{AVSS}}{\epsilon_{AVSS} \cdot d}$	K_{O_2}	0.5	$\frac{\epsilon_{O_2}}{m^3}$
Y_H	0.4	$\frac{\epsilon_{HVSS}}{\epsilon_{bCOD}}$	Y_A	0.12	$\frac{\epsilon_{AVSS}}{\epsilon_{NH_3N}}$	$F_{N.cells}$	168	$\frac{\epsilon_N}{\epsilon_{VSS}}$
K_S	20	$\frac{\epsilon_{COD}}{m^3}$	K_N	0.74	$\frac{\epsilon_{NH_3N}}{m^3}$	$F_{O_2.VSS}$	1374	$\frac{\epsilon_{O_2}}{\epsilon_{VSS}}$
k_{DH}	0.12	$\frac{\epsilon_{HVSS}}{\epsilon_{HVSS} \cdot d}$	k_{DA}	0.08	$\frac{\epsilon_{AVSS}}{\epsilon_{AVSS} \cdot d}$	$F_{O_2.NH_3}$	4.57	$\frac{\epsilon_{O_2}}{\epsilon_{VSS}}$
f_H	0.15	$\frac{\epsilon_{HCD}}{\epsilon_{HVSS}}$	f_A	0.15	$\frac{\epsilon_{ACD}}{\epsilon_{AVSS}}$			$\frac{\epsilon_{O_2}}{\epsilon_{NH_3N}}$

RAS characterization

$$\begin{pmatrix} F_{H.R} \\ F_{A.R} \\ F_{HCD.R} \\ F_{ACD.R} \\ F_{FSS.R} \\ F_{nbVSS.R} \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.02 \\ 0.07 \\ 0.01 \\ 0.06 \\ 0.06 \end{pmatrix} \begin{pmatrix} X_{R.H} \\ X_{R.A} \\ X_{R.HCD} \\ X_{R.ACD} \\ X_{R.FSS.R} \\ X_{R.nbVSS.R} \end{pmatrix} = X_R \begin{pmatrix} F_{H.R} \\ F_{A.R} \\ F_{HCD.R} \\ F_{ACD.R} \\ F_{FSS.R} \\ F_{nbVSS.R} \end{pmatrix} = \begin{pmatrix} 7.8 \times 10^3 \\ 200 \\ 700 \\ 100 \\ 600 \\ 600 \end{pmatrix} \frac{\text{g}}{\text{m}^3}$$

Reactor parameters

$$Q_{MGD} := 9.985$$

$$\begin{pmatrix} H \\ W_{CH} \\ Q \\ V_R \\ D_d \\ n_z \end{pmatrix} := \begin{pmatrix} 5 \\ 8.366 \\ \frac{Q_{MGD} \cdot 10^6}{3.2808^3 \cdot 7.48} \\ 6300 \\ 0.11945 \\ 100 \end{pmatrix} = \begin{pmatrix} 5 \\ 8.366 \\ 3.78 \times 10^4 \\ 6.3 \times 10^3 \\ 0.119 \\ 100 \end{pmatrix} \begin{pmatrix} m \\ m \\ \frac{m^3}{d} \\ m^3 \\ \frac{m^2}{s} \\ \text{segments} \end{pmatrix}$$

Somewhat greater than found by Chambers and Jones (1988)

Reactor Loading parameters

$$\begin{pmatrix} \text{COD}_{in} \\ N_{TK.in} \\ N_{NH3.in} \\ X_{HVSS.in} \\ X_{AVSS.in} \\ FSS_{in} \\ nbVSS_{in} \end{pmatrix} = \begin{pmatrix} 300 \\ 50 \\ 20 \\ 1 \\ .1 \\ 5 \\ 5 \end{pmatrix} \begin{pmatrix} \frac{E_{COD}}{m^3} \\ \frac{E_{TKN}}{m^3} \\ \frac{E_{NH3N}}{m^3} \\ \frac{E_{HVSS}}{m^3} \\ \frac{E_{AVSS}}{m^3} \\ \frac{E_{FSS}}{m^3} \\ \frac{E_{nbVSS}}{m^3} \end{pmatrix}$$

Further definitions and the performance objective

$$N_{\text{org.in}} := N_{\text{TK.in}} - N_{\text{NH3.in}} \quad F_{\text{N.org}} := \frac{N_{\text{org.in}}}{\text{COD}_{\text{in}}} = 0.1$$

$$\text{DO} := 1 - \frac{\xi_{\text{O2}}}{3}$$

$$N_{\text{NH3.target}} := 1 - \frac{\xi_{\text{NH3N}}}{3}$$

Base solve block for the single CMFR and CMFRS in Series models

Given

$$\frac{S_{in} - S}{\tau_N} = \frac{\mu_{max,H} S \cdot X_H}{Y_H (K_S + S)}$$

$$\frac{X_H - X_{H,in}}{\tau_N} = \left(\frac{\mu_{max,H} S}{K_S + S} - k_{DH} \right) \cdot X_H$$

$$X_{HCD} = X_{HCD,in} + \tau_N f_H \cdot k_{DH} X_H$$

$$X_{ACD} = X_{ACD,in} + \tau_N f_A \cdot k_{DA} X_A$$

$$\frac{OrgN_{in} - N_{org}}{\tau_N} = F_{N,org} \cdot \frac{\mu_{max,H} S \cdot X_H}{Y_H (K_S + S)}$$

$$\frac{X_A - X_{A,in}}{\tau_N} = \left(\frac{\mu_{max,A} \cdot NH_3N}{K_N + NH_3N} \cdot \frac{DO}{K_{O_2} + DO} - k_{DA} \right) \cdot X_A$$

$$0 = \frac{NH_3N_{in} - NH_3N}{\tau_N} + F_{N,org} \cdot \frac{\mu_{max,H} S \cdot X_H}{Y_H (K_S + S)} - \frac{\mu_{max,A} \cdot NH_3N \cdot X_A}{Y_A \cdot (K_N + NH_3N)} \cdot \frac{DO}{K_{O_2} + DO}$$

$$- F_{N,cells} \cdot \left[\left(\frac{\mu_{max,H} S}{K_S + S} - k_{DH} \right) \cdot X_H + f_H \cdot k_{DH} X_H + \left(\frac{\mu_{max,A} \cdot NH_3N}{K_N + NH_3N} \cdot \frac{DO}{K_{O_2} + DO} - k_{DA} \right) \cdot X_A + f_A \cdot k_{DA} X_A \right]$$

$$Tis(\tau_N, S_{in}, X_{H,in}, OrgN_{in}, NH_3N_{in}, X_{A,in}, X_{HCD,in}, X_{ACD,in}, S, X_H, N_{org}, NH_3N, X_A, X_{HCD}, X_{ACD})$$

$$:= Find(S, X_H, N_{org}, NH_3N, X_A, X_{HCD}, X_{ACD})$$

Set of seven ODEs for the PFR and SF models

$$\begin{array}{l}
 \frac{d(cbCOD)}{d\tau} \\
 \frac{dX_{HVSS}}{d\tau} \\
 \frac{d(OrgN)}{d\tau} \\
 \frac{dX_{AVSS}}{d\tau} \\
 \frac{dX_{HCD}}{d\tau} \quad \frac{dX_{ACD}}{d\tau} \\
 \frac{dNH_3N}{d\tau}
 \end{array}
 \mathbf{D}(t, \mathbf{y}) :=
 \begin{array}{l}
 \frac{-\mu_{max,H} y_0 y_1}{Y_H (K_S + y_0)} \\
 \left(\frac{\mu_{max,H} y_0}{K_S + y_0} - k_{DH} \right) y_1 \\
 F_{N,org} \frac{-\mu_{max,H} y_0 y_1}{Y_H (K_S + y_0)} \\
 \left(\frac{\mu_{max,A} y_6}{K_N + y_6} \frac{DO}{K_{O_2} + DO} - k_{DA} \right) y_3 \\
 f_H k_{DH} y_1 \\
 f_A k_{DA} y_3 \\
 F_{N,org} \frac{\mu_{max,H} y_0 y_1}{Y_H (K_S + y_0)} - F_{N,cells} \left[\left(\frac{\mu_{max,H} y_0}{K_S + y_0} - k_{DH} \right) y_1 + \left(\frac{\mu_{max,A} y_6}{K_N + y_6} \frac{DO}{K_{O_2} + DO} - k_{DA} \right) y_3 + f_H k_{DH} y_1 + f_A k_{DA} y_3 \right] - \frac{\mu_{max,A} y_6}{Y_A (K_N + y_6)} \frac{DO}{K_{O_2} + DO} y_3
 \end{array}
 \begin{array}{l}
 y_0 \\
 y_1 \\
 y_2 \\
 y_3 \\
 y_4 \\
 y_5 \\
 y_6
 \end{array}
 =
 \begin{array}{l}
 S \\
 X_H \\
 OrgN \\
 X_A \\
 X_{HCD} \\
 X_{ACD} \\
 NH_3N
 \end{array}$$

S_{PFR}
 $X_{H,PFR}$
 $N_{org,PFR}$
 $X_{A,PFR}$
 $X_{HCD,PFR}$
 $X_{ACD,PFR}$
 $N_{NH3,PFR}$
 SRT_{PFR}
 $MLSS_{PFR}$
 R_{PFR}

$$\begin{pmatrix} R \\ \text{count} \\ \Delta R \end{pmatrix} \leftarrow \begin{pmatrix} 0 \\ 0 \\ 0.01 \end{pmatrix}$$

```

while  $N_{NH3.out\_count} > N_{NH3.target}$ 
  count  $\leftarrow$  count + 1
  R  $\leftarrow$  R +  $\Delta R$ 

```

$$y \leftarrow \frac{1}{1+R} \begin{pmatrix} R \cdot S_{out_count-1} + COD_{in} \\ R \cdot X_{R,H} \\ R \cdot N_{org,out_count-1} + N_{org,in} \\ R \cdot X_{R,A} \\ R \cdot X_{R,HCD} \\ R \cdot X_{R,ACD} \\ R \cdot N_{NH3,out_count-1} + N_{NH3,in} \end{pmatrix}$$

$$\tau_{SM,R} \leftarrow \frac{\tau_{nom}}{1+R}$$

$$Sol \leftarrow \text{rkfixed}(y, 0, \tau_{SM,R}, n_z, D)$$

for $k \in 0..n_z$

$$\begin{pmatrix} S_k \\ X_{H_k} \\ N_{org_k} \\ X_{A_k} \\ X_{HCD_k} \\ X_{ACD_k} \\ N_{NH3_k} \end{pmatrix} \leftarrow \begin{pmatrix} (Sol \langle 1 \rangle)_k \\ (Sol \langle 2 \rangle)_k \\ (Sol \langle 3 \rangle)_k \\ (Sol \langle 4 \rangle)_k \\ (Sol \langle 5 \rangle)_k \\ (Sol \langle 6 \rangle)_k \\ (Sol \langle 7 \rangle)_k \end{pmatrix}$$

$$R_{gex} \leftarrow Q(1+R) \cdot (X_{H_{n_z}} - X_{H_0} + X_{A_{n_z}} - X_{A_0} + X_{HCD_{n_z}} - X_{HCD_0} + X_{ACD_{n_z}} - X_{ACD_0})$$

$$BI \leftarrow \frac{V_R}{2 \cdot n_z} \sum_{k=1}^{n_z} (X_{H_k} + X_{H_{k-1}} + X_{A_k} + X_{A_{k-1}} + X_{HCD_k} + X_{HCD_{k-1}} + X_{ACD_k} + X_{ACD_{k-1}})$$

$$SRT_{count} \leftarrow \frac{BI}{R_{gex}}$$

$$MLSS_{count} \leftarrow X_{H_{n_z}} + X_{A_{n_z}} + X_{HCD_{n_z}} + X_{ACD_{n_z}} + \frac{SRT_{count}}{\tau_{nom}} (FSS_{in} + nbVSS_{in})$$

$$\begin{pmatrix} S_{out_count} \\ N_{NH3.out_count} \\ N_{org.out_count} \end{pmatrix} \leftarrow \begin{pmatrix} S_{n_z} \\ N_{NH3_{n_z}} \\ N_{org_{n_z}} \end{pmatrix}$$

return

$$\begin{pmatrix} S \\ X_H \\ N_{org} \\ X_A \\ X_{HCD} \\ X_{ACD} \\ N_{NH3} \\ SRT \\ MLSS \\ R \end{pmatrix}$$

$Sol \leftarrow \text{rkfixed}(y, 0, \tau_{SM,R}, n_z, D)$

Model Output

parameter	Reactors			
	CMFR	TiS		
cbCOD	single values for mixed influent and effluent for each parameter for each R	N+1 values for each parameter for each R	$(n_\tau + 1) \times (n_z + 1)$ matrix for each parameter for each R	
OrgN				
NH ₃ N				
X _{HVSS}				
X _{AVSS}				
X _{HCD}				
X _{ACD}				
Γ^{GHVSS}	single value for each upon completion of iteration	N values for each upon completion of iteration	$(n_\tau) \times (n_z)$ matrix for each rate upon completion of iteration	
Γ^{GHVSS}				
Γ^{GHCD}				
Γ^{GACD}				
$\Gamma^{\text{OrgN} \rightarrow \text{NH}_3\text{N}}$				
$\Gamma^{\text{NH}_3\text{N} \rightarrow \text{NO}_3\text{N}}$				
$\Gamma^{\text{N} \rightarrow \text{VSS}}$				
Γ^{O_2}	single value at each iteration			
R (recycle ratio)	single value at each iteration			
MLVSS	single value at each iteration			
MLSS	single value at each iteration			
SRT	single value at each iteration			
R _{O2}	single value upon completion of iteration			
N	N/A	single value at each iteration	N/A	single value at each iteration
E(τ) vs τ	N/A	N/A	N/A	n_τ values at each iteration
τ	single value at each iteration	N values at each iteration	single value at each iteration	

Profiles of parameter values

cbCOD, OrgN, NH₃-N:

- integrate numerically across the parallel reactors using the discrete values for each specified τ .

$$C_z = \frac{h}{3} \sum_{j=j_{beg}}^{j_{end}} SR_j E(\tau_j) C_{j,z} \quad h = \tau_{j_{beg}+1} - \tau_{j_{beg}}$$

$E(\tau)$ vs τ ordered pairs are preserved from the last iteration

SR is the set of Simpson's 1/3 rule coefficients:

1, 4, 2, 4, 2, ... , 4, 2, 4, 1

corresponding to the n_τ $E(\tau)$ vs τ pairs.

Profiles of process rates

r_{cbCOD}
 $r_{OrgN \rightarrow NH_3N}$
 $r_{N \rightarrow VSS}$
 $r_{NH_3N \rightarrow NO_3N}$
 r'_{HVSS}
 r'_{AVSS}
 r_{HCD}
 r_{ACD}
 r_{O_2}

Fraction of the total reactor volume of reactor with $\tau = \tau_j$.

$$r_z = \frac{h}{3} \sum_{j=j_{beg}}^{j_{end}} SR_j \frac{\tau_j}{\bar{\tau}_R} E(\tau_j) r_{j,z}$$

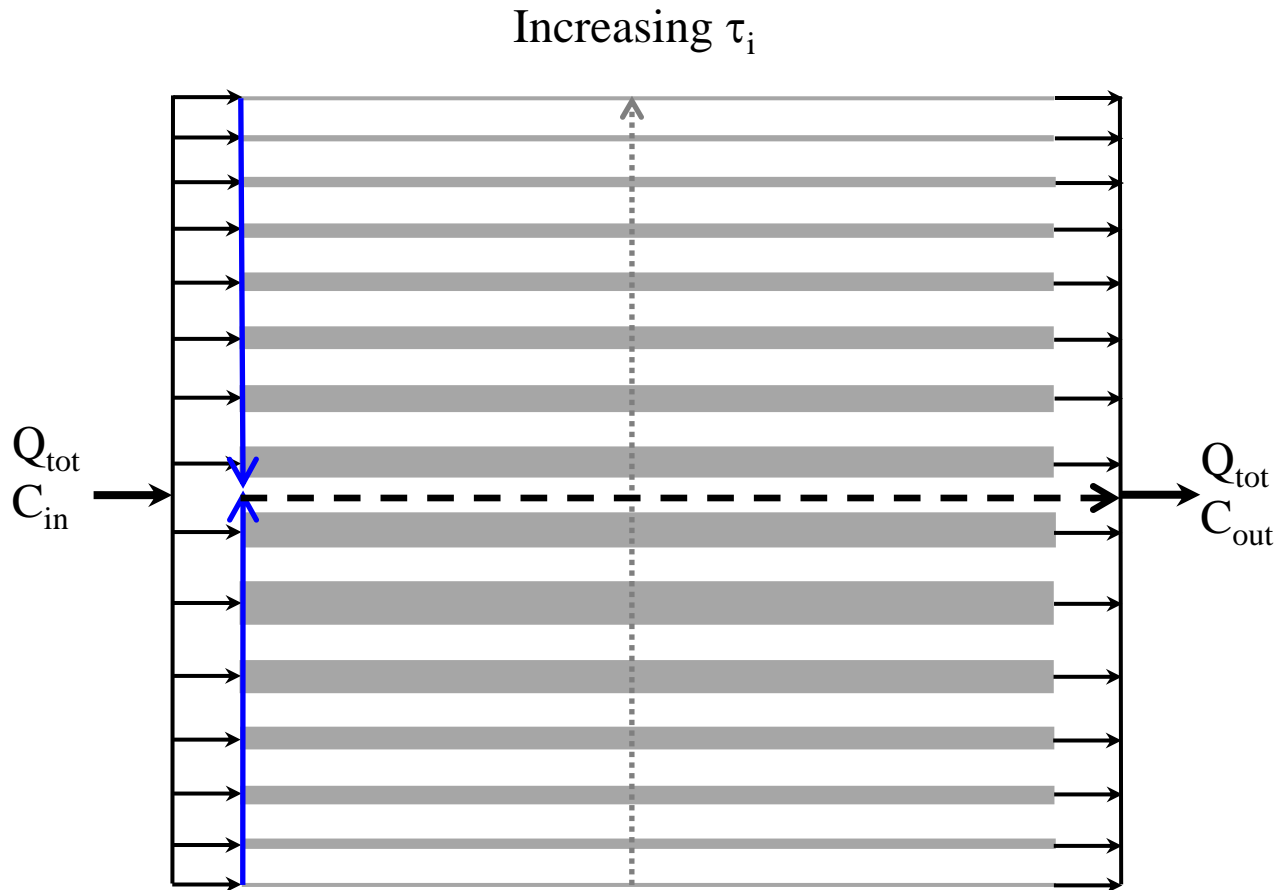
Developed for the SF model by Mott (2014)

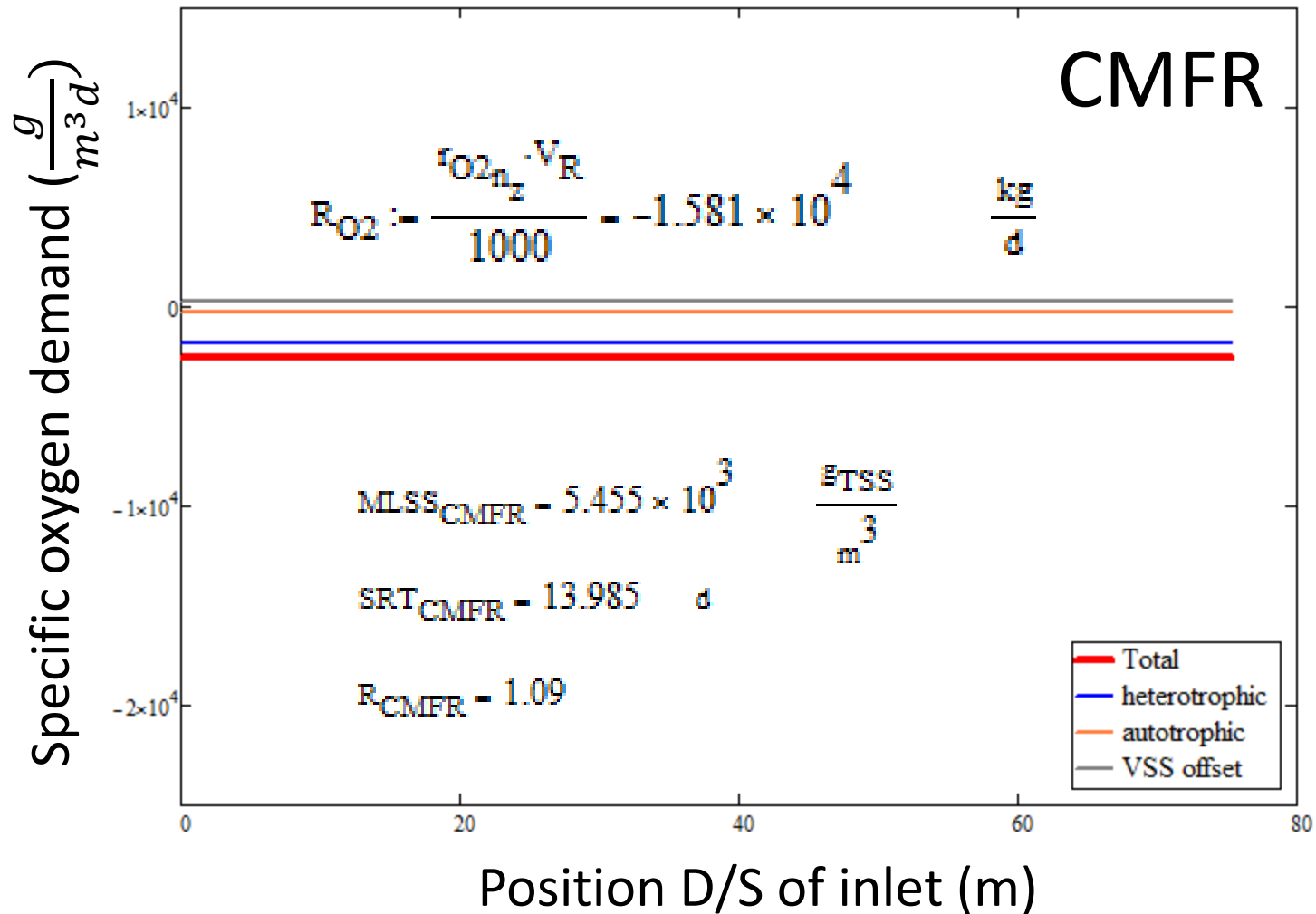
```

function profiles(rz)
    for k = 0..nz
        [rz.O2_k; rz.SU_k; rz.GX_k; rz.NO3N_k] ← h/3 * sum_{j=j_beg}^{j_end} [ E_tau_j * (tau_SF_j / (tau_nom * (1 + R_SF))) * SR_j * [ rz.O2_j,k; rz.SU_j,k; rz.GX_j,k; rz.NO3N_j,k ] ]
    return [rz.O2; rz.SU; rz.GX; rz.NO3N]
    
```

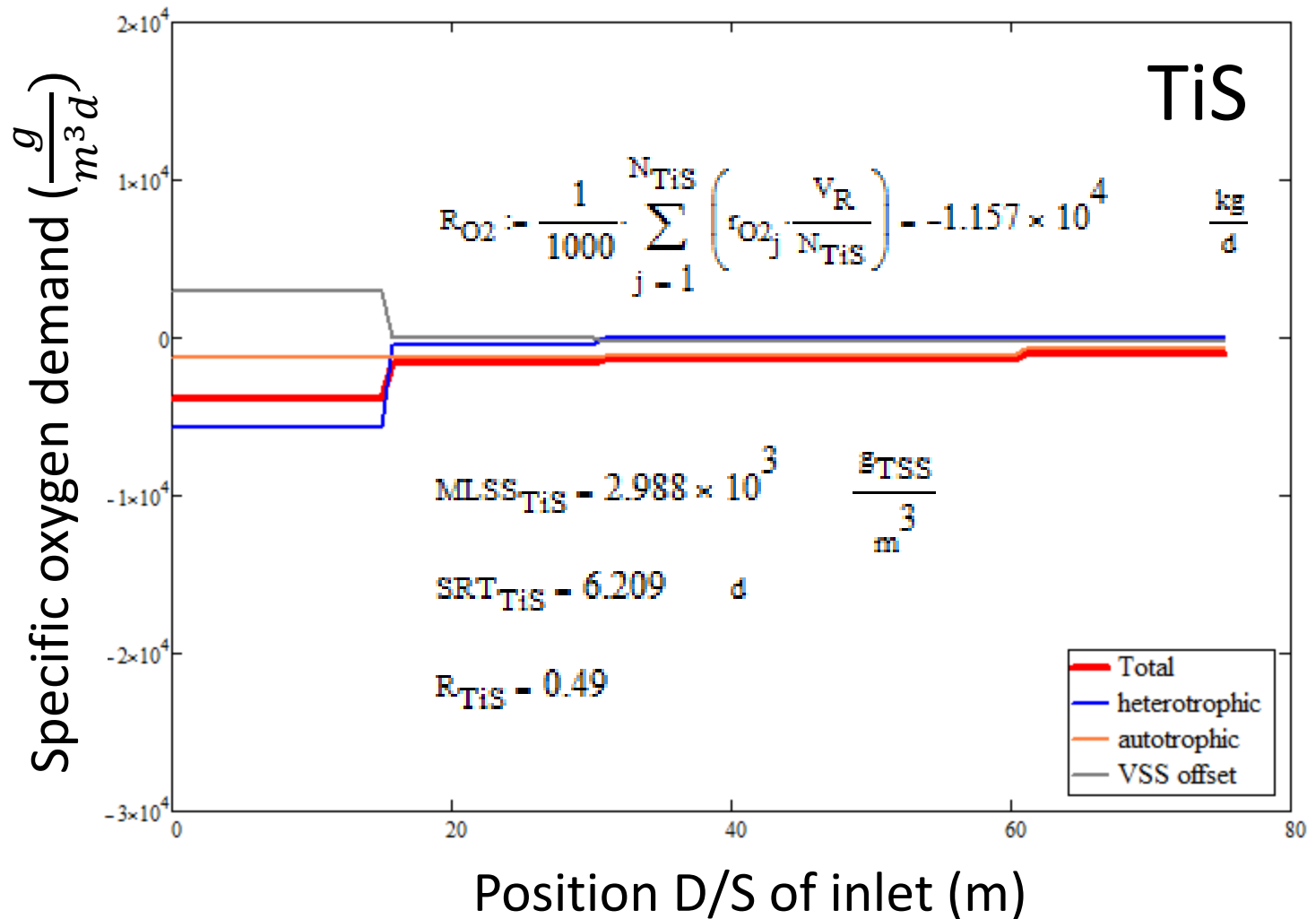
The Segregated Flow Model:

A set of parallel PFRs comprises the reactor

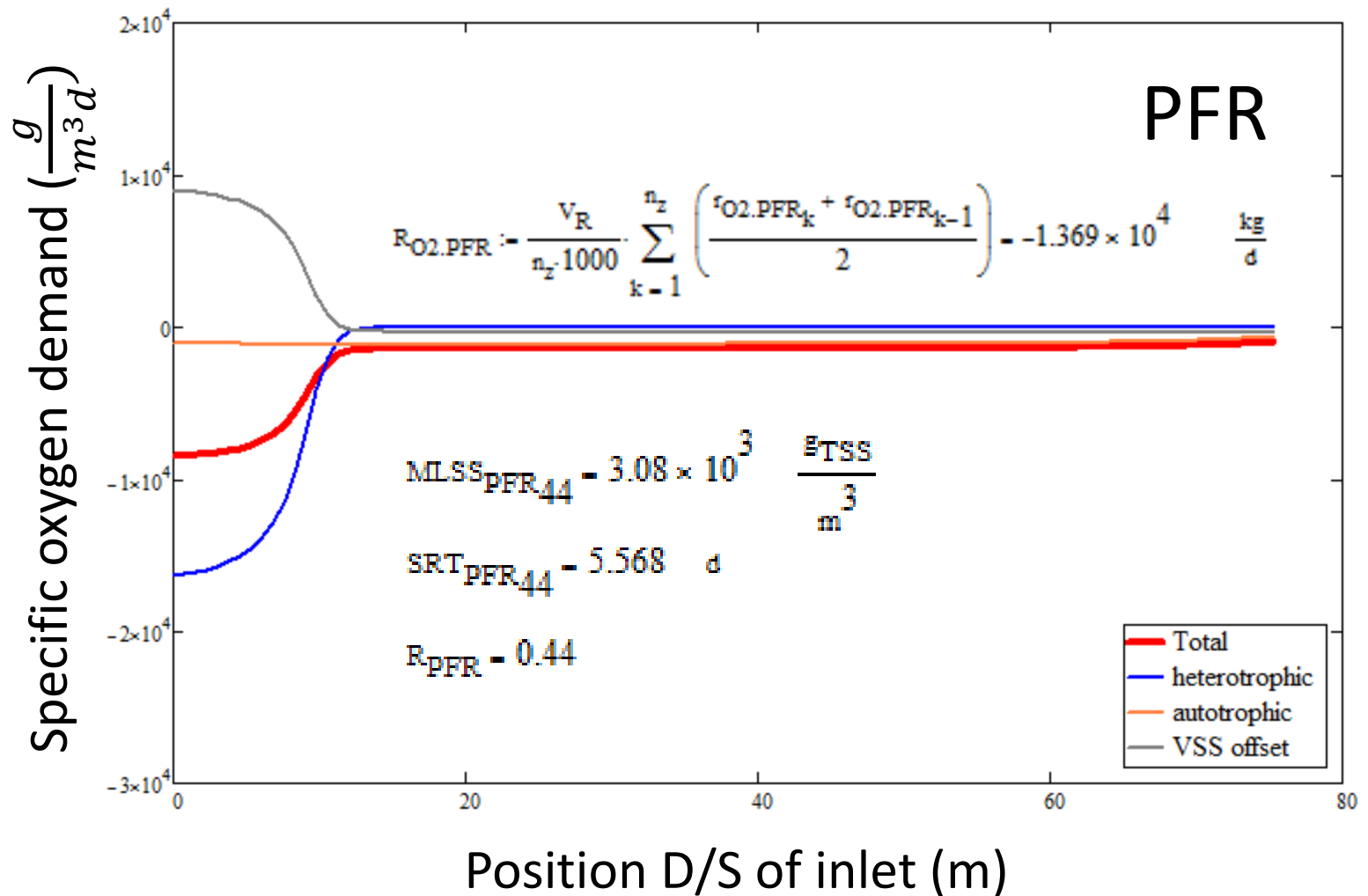




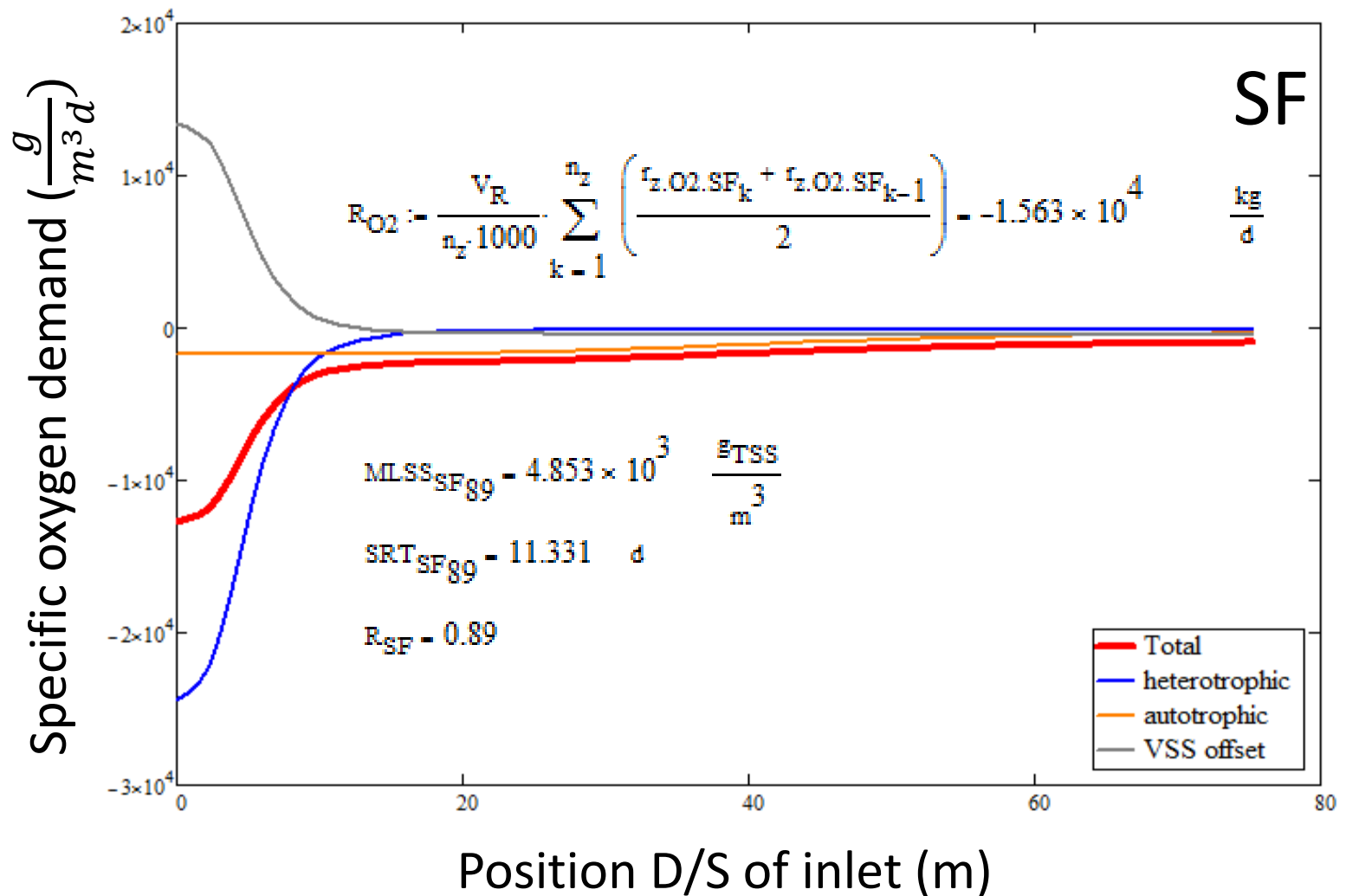
Specific oxygen utilization (and its components) along the flow path of a hypothetical reactor modeled using the single CMFR model.



Specific oxygen utilization (and its components) along the flow path of a hypothetical reactor modeled using the CMFRs in Series model.

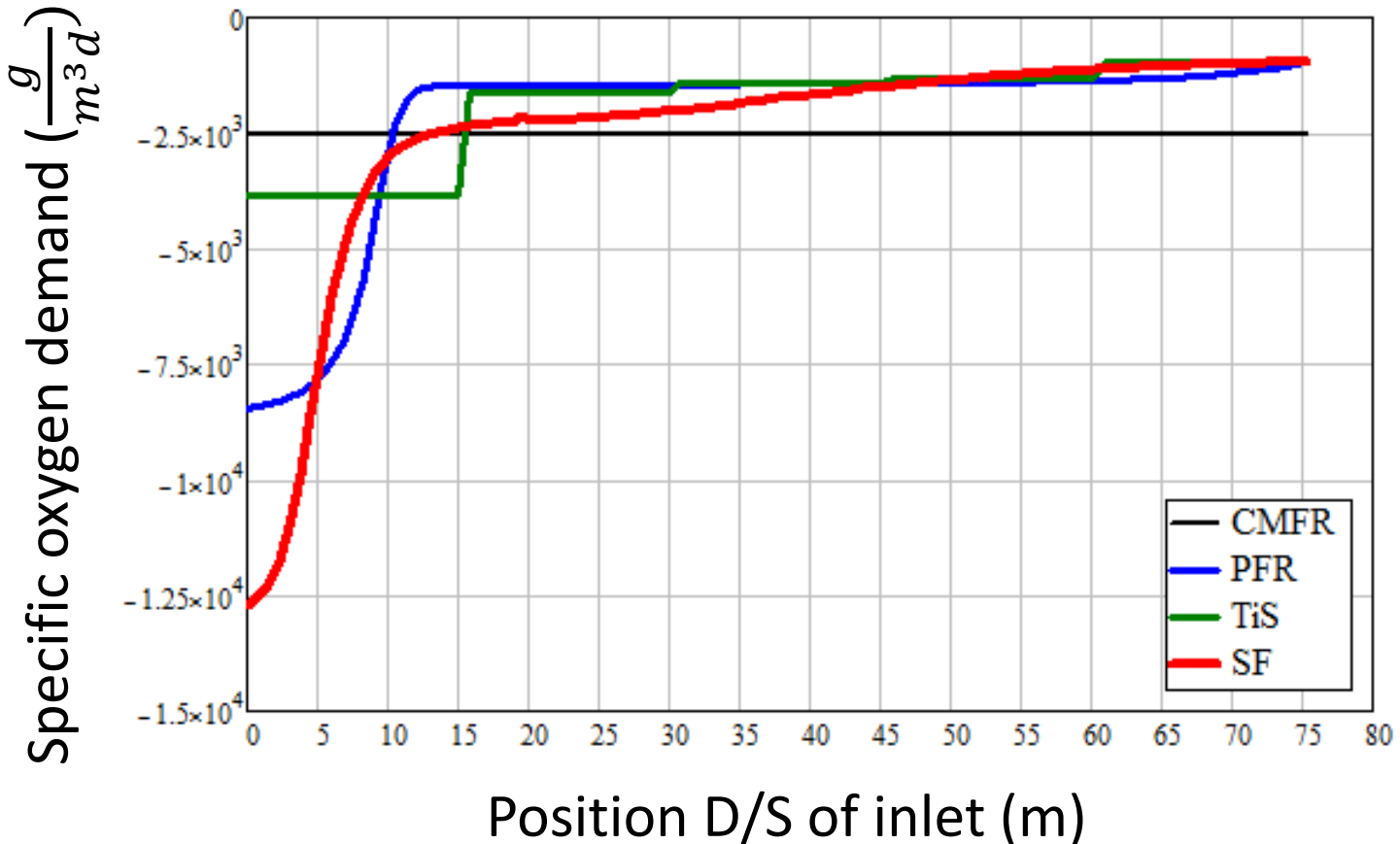


Specific oxygen utilization (and its components) along the flow path of a hypothetical reactor modeled using the PFR model.



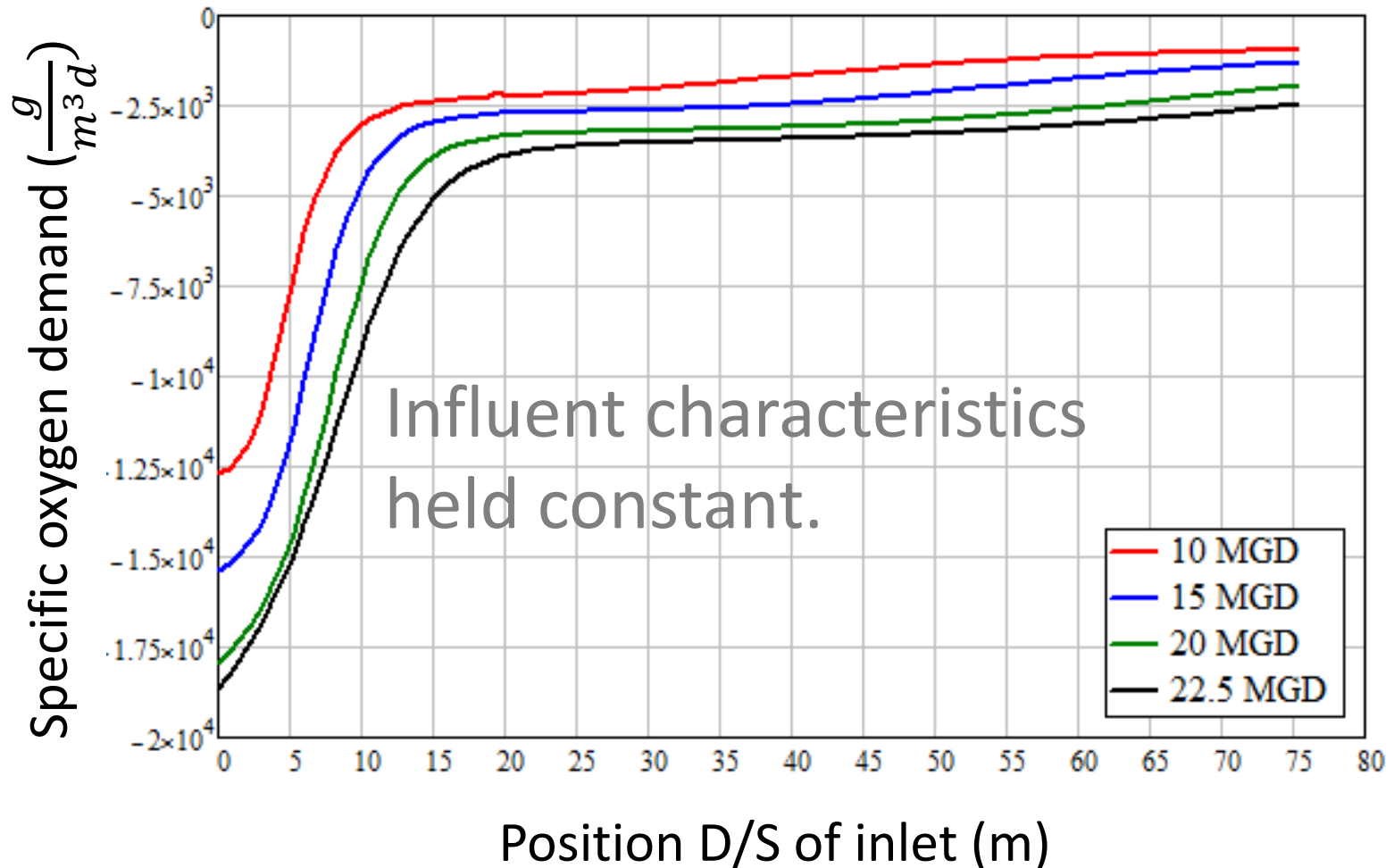
Specific oxygen utilization (and its components) along the flow path of a hypothetical reactor modeled using the segregated flow model.

Comparisons of predictions for specific oxygen utilization from the four reactor models.



Specific oxygen utilization along the flow path of a hypothetical reactor modeled using single CMFR, CMFRs in Series, PFR, and SF models.

Specific oxygen utilization predicted from SF for additional process loadings



Specific oxygen utilization along the flow path of a hypothetical reactor modeled using the SF model for varying loadings.

MathCAD programming to convert $r_{O_2,z}$ to numbers of diffusers per supply lateral – Max case is illustrated.

Diffuser investigated has flow range from 0.5 – 10 SCFM/ft²_{diff}
 Mass of oxygen delivered per diffuser per day

Must solve a set of three equations to obtain AOTE

Actual OTR per diffuser

Required number of diffusers per m of L_{FP}

Required number of diffusers per lateral along L_{FP}

$$\begin{pmatrix} \text{SOTE} \\ V_{\text{Air,diff}} \\ A_{\text{diff}} \\ R_{\text{O}_2,\text{max}} \end{pmatrix} := \begin{pmatrix} 0.24 \\ 7.5 \\ 1 \\ -3.191 \times 10^4 \end{pmatrix} \quad \begin{pmatrix} \frac{\text{kg}_{\text{tr}}}{\text{kg}_{\text{app}}} \\ \frac{f_{\text{std}}^3}{\text{min} \cdot f_{\text{diff}}^2} \\ \frac{f_{\text{diff}}^2}{\text{diff}} \\ \frac{\text{kg}_{\text{O}_2}}{d} \end{pmatrix} \quad \begin{pmatrix} R_{\text{atm}} \\ Y_{\text{O}_2} \\ T_{\text{std}} \\ P_{\text{std}} \\ \text{MW}_{\text{O}_2} \end{pmatrix} := \begin{pmatrix} 0.000082057 \\ 0.209 \\ 293 \\ 1 \\ 0.032 \end{pmatrix} \quad \begin{pmatrix} \frac{3}{\text{m} \cdot \text{atm}} \\ \frac{\text{mol}_{\text{O}_2}}{\text{mol}_{\text{air}}} \\ \text{°K} \\ \text{atm} \\ \frac{\text{kg}}{\text{mol}} \end{pmatrix}$$

$$V_{\text{Air,diff}} := \frac{V_{\text{Air,diff}}}{A_{\text{diff}} \cdot 3.2808^3} \cdot 1440 = 305.833 \frac{\text{m}_{\text{air}}^3}{\text{diff} \cdot d} \quad V_{\text{O}_2,\text{diff}} := V_{\text{Air,diff}} \cdot Y_{\text{O}_2} = 63.919 \frac{\text{m}_{\text{O}_2}^3}{\text{diff} \cdot d}$$

$$M_{\text{O}_2,\text{diff}} := V_{\text{O}_2,\text{diff}} \cdot \frac{P_{\text{std}}}{R_{\text{atm}} \cdot T_{\text{std}}} \cdot \text{MW}_{\text{O}_2} = 85.074 \frac{\text{kg}_{\text{O}_2}}{\text{diff} \cdot d}$$

$$\text{AOTE} = \text{SOTE} \cdot \left(\frac{\beta \cdot C_{s,\text{bar}} - C_L}{C_s} \right) \cdot 1.024^{T-20} \cdot \alpha \cdot F \quad \text{AOTE} := 0.16 \frac{\text{kg}_{\text{O}_2,\text{tr}}}{\text{kg}_{\text{O}_2,\text{app}}}$$

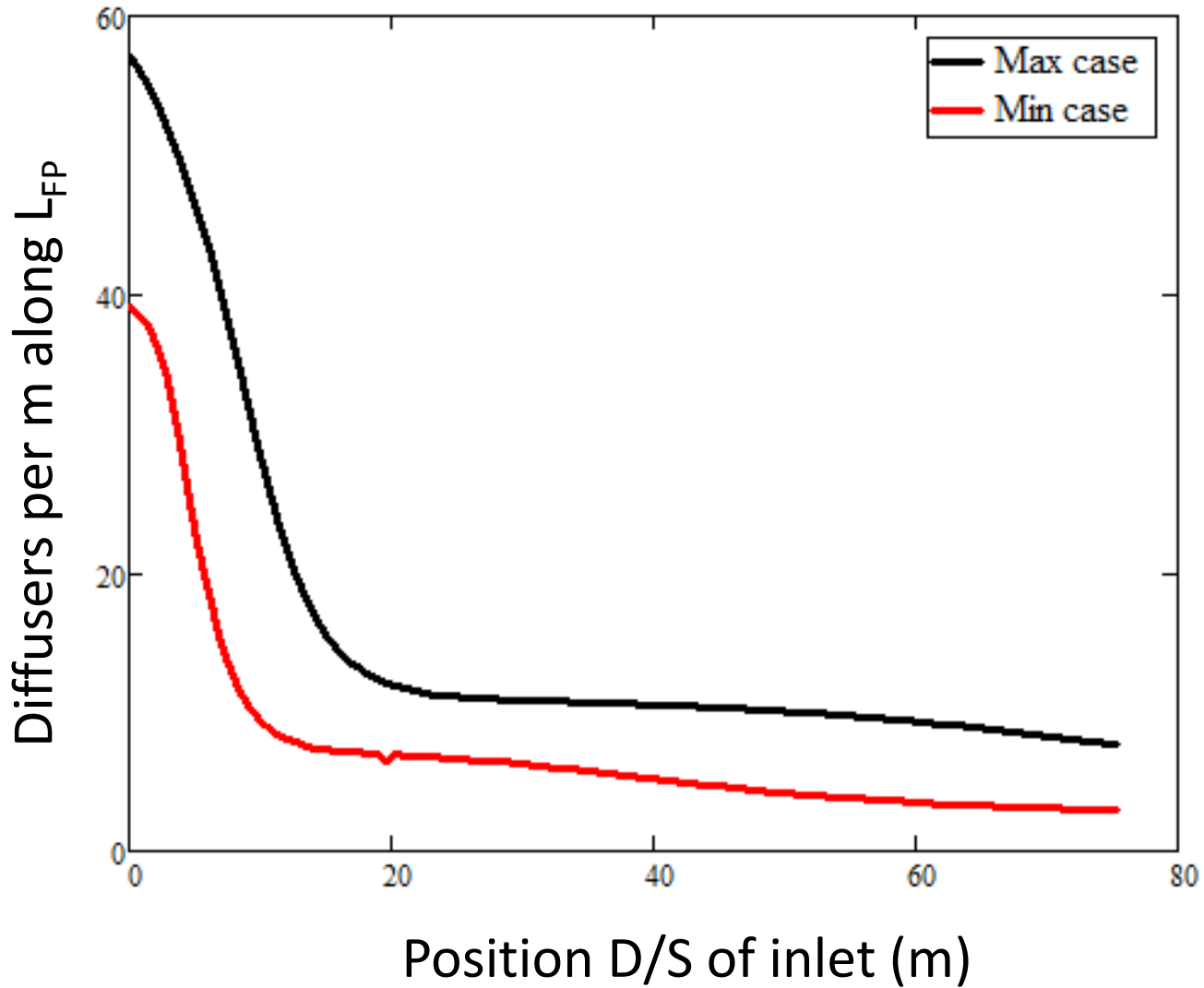
$$\text{AOTR}_{\text{diff}} := M_{\text{O}_2,\text{diff}} \cdot \text{AOTE} = 13.612 \quad N_{\text{diff}} := \frac{-R_{\text{O}_2,\text{max}}}{\text{AOTR}_{\text{diff}}} = 2.344 \times 10^3$$

$$r_{\text{O}_2,\text{max}} := \frac{r_{z,\text{O}_2,\text{SF},22.5}}{1000} \cdot \frac{\text{kg}_{\text{O}_2}}{\text{m}^3 \cdot d} \quad R_{\text{O}_2,z} := r_{\text{O}_2,\text{max}} \cdot A_X \cdot \frac{\text{kg}_{\text{O}_2}}{\text{m} \cdot d}$$

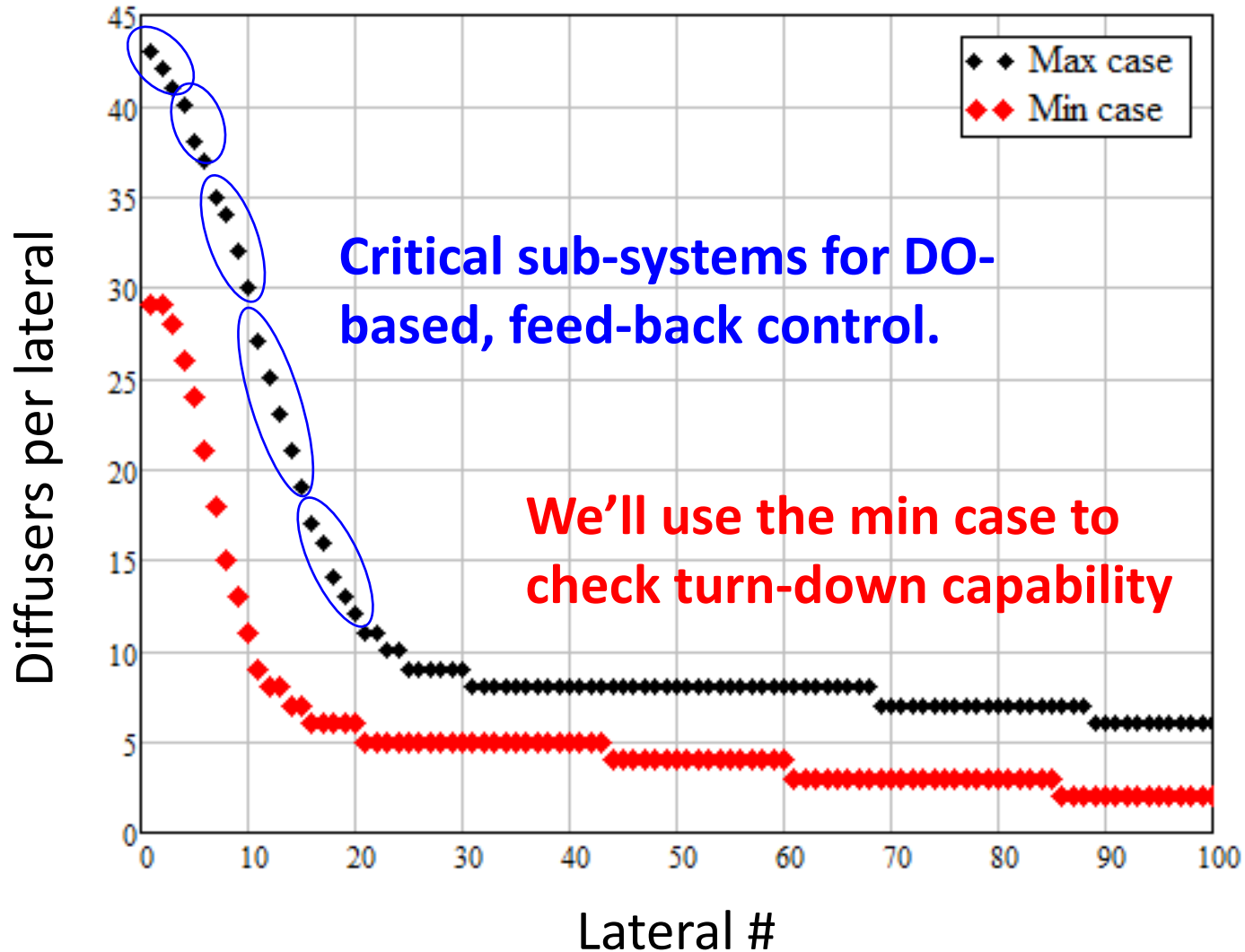
$$\text{diff}_{\text{dens}} := \frac{-R_{\text{O}_2,z}}{\text{AOTR}_{\text{diff}}} \cdot \frac{\text{diff}}{\text{m}}$$

```
diff_dens.2 := for k ∈ 1..n2 Assume 100 laterals spaced ~3/4 m apart.
diff_dens.2_k ← round( (L_FP * diff_dens_k + diff_dens_{k-1}) / n2 )
return diff_dens.2
```

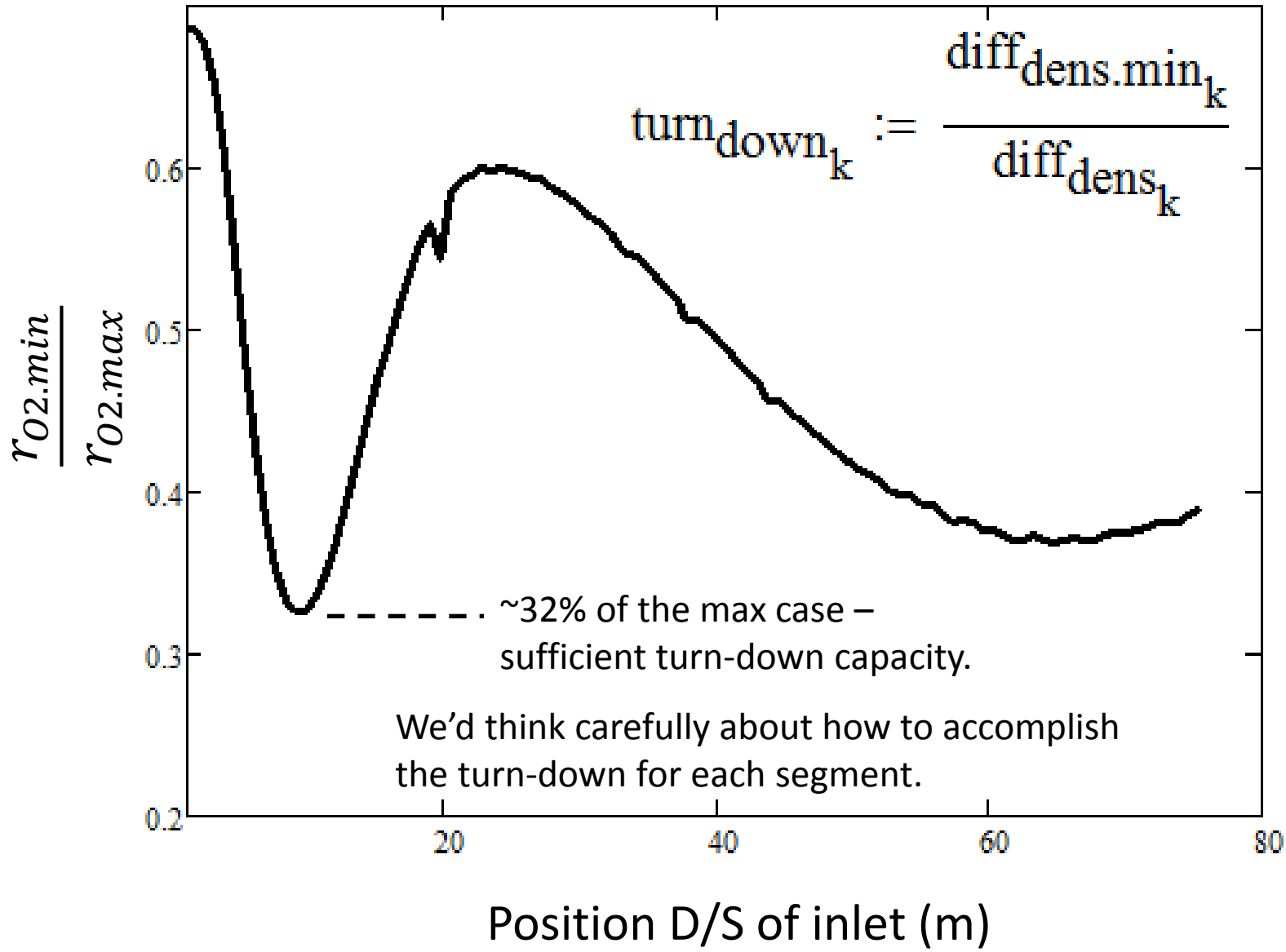
$$N_{\text{diff},2} := 2 \cdot \sum_{k=1}^{n_2} \text{diff}_{\text{dens},2_k} = 2.342 \times 10^3$$



Predicted diffuser density (diffusers per m) along the reactor flow path



Predicted number of diffusers per lateral - assuming 100 laterals spaced at ~ 0.75 m intervals along L_{FP} .



Some weaknesses: (additional work to do)

- Soluble and insoluble organics manifest as cbCOD have different biodegradability
- Process parameters for rates and yield are uncertain
- RAS characterization is uncertain
 - variable in concentration and composition
 - dependent on process performance
 - dependent on organic substrate character
- D_d itself is spatially variant
 - correlated with aeration system configuration
 - specific to aeration diffusers (bubble size)
 - dependent upon air flow to diffusers
- Other weaknesses ...

Thank you for your attention.

Questions?

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